CT IMAGING

- MACHINES
- RADON TRANSFORM
- FOURIER SLICE THEOREM
- FILTERED BACKPROJECTION
- FAN-BEAM CT
- CONE-BEAM & MISC.

IDEA

- EMIT X-RAY ON PATH THROUGH OBJECT
- MEASURE TOTAL ATTENUATION
- DO THIS FOR MANY RAY
- BACKPROJECT

[FIG 5.32, 5.33, 5.34]

CT MACHINES

GEN 1: PENCIL BEAM SWEPT LINEARLY AT EACH ANGLE

GEN 2: FAN BEAM SWEPT LINEARLY AT EACH ANGLE

GEN 3: WIDE FAN BEAM COVERING WHOLE OF OBJECT SWEPT RADIALLY

GEN 4: CIRCULAR RING OF DETECTORS WITH ONLY SOURCE ROTATING

ALSO: CONE-BEAM CT
HELICAL CT
MULTISLICE CT

[FIG 5.35]

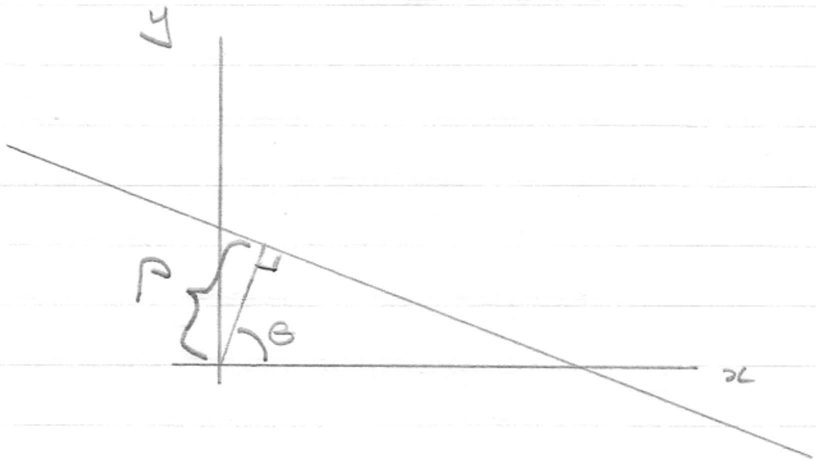
RADON TRANSFORM

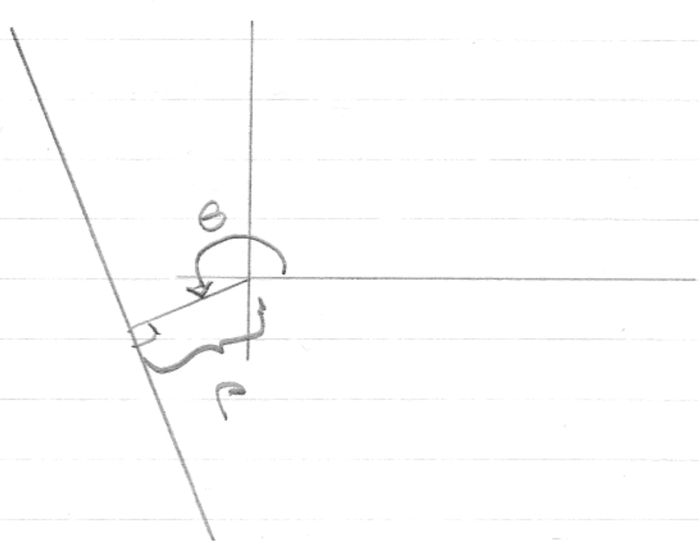
LINES IN 2D SPACE

$y = mx + b$ M, b DEFINE LINE

— OR —

$x \cos\theta + y \sin\theta = \rho$ θ, ρ DEFINE LINE





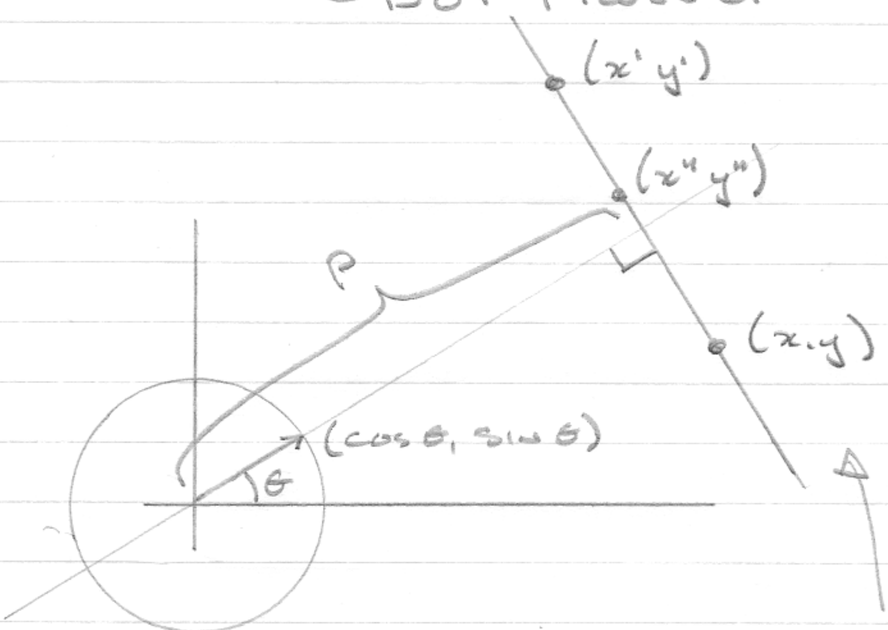
INTUITION:

$$x \cos\theta + y \sin\theta = \rho$$

$$(x,y) \cdot (\cos\theta, \sin\theta) = \rho$$

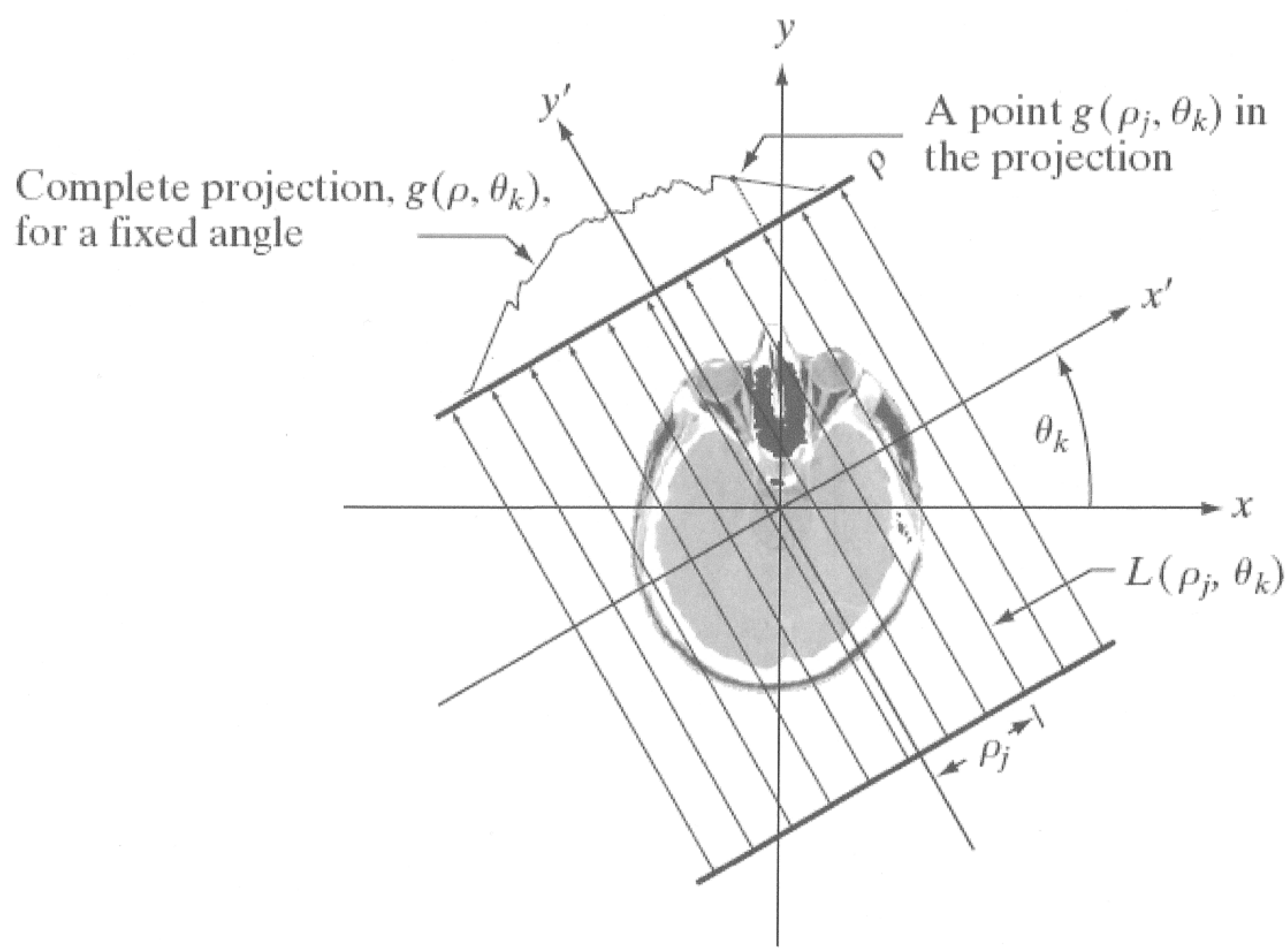
VECTOR ON UNIT CIRCLE

DOT PRODUCT



ALL OF THESE VECTOR HAVE SAME DOT PRODUCT WITH $(\cos\theta, \sin\theta)$.

THAT DOT PRODUCT IS THE DISTANCE OF THE PROJECTION OF (x,y) ONTO THE LINE OF $(\cos\theta, \sin\theta)$ $= \rho$



PARALLEL-BEAM GEOMETRY

[GONZALES + WOODS]

PARALLEL-BEAM PROJECTION

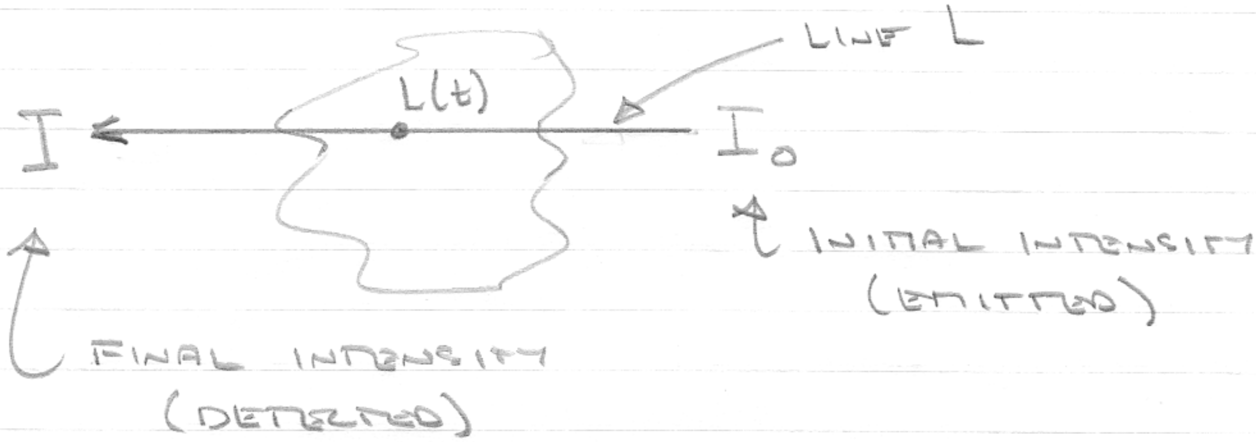
- ALL BEAMS PARALLEL
- DETECTOR AT ANGLE θ_k
- $g(\rho, \theta_k)$ IS THE PROJECTION AT ANGLE θ_k

  ρ IS IN A LIMITED RANGE $[-R, +R]$

  SAMPLES $\rho_0\ \rho_1\ \rho_2 \cdots \rho_{N-1}$ ARE EVENLY SPACED AND FROM PARALLEL PROJECTION LINES

- $f(x,y)$ IS THE X-RAY ATTENUATION DENSITY AT (x,y)

ATTENUATION



$$I = I_0 \, e^{-\int_L f(L(t))\,dt}$$

f = X-RAY ABSORPTION AT $L(t)$ (IN $\frac{\%\ \text{ABSORBED}}{\text{DISTANCE}}$)

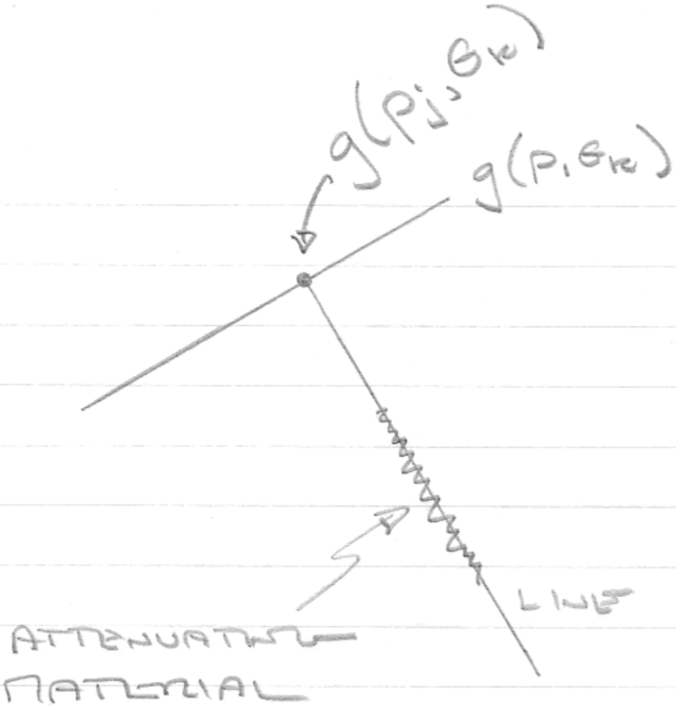
$$\log I = \log I_0 - \int_L f(L(t))\,dt$$

$$\int_L f(L(t))\,dt = \log I_0 - \log I$$

CALCULATED

EMITTED

RECEIVED



LINE $L(p_j, \theta_k)$;

$$x \cos\theta_k + y \sin\theta_k = p_j$$

$$g(p_j, \theta_k) = \int_{L(p_j \theta_k)} f(L(t))\,dt$$

$$= \int_y \int_x f(x,y)\ \underbrace{\delta(x\cos\theta_k + y\sin\theta_k - p_j)}_{=1 \text{ IFF } (x,y) \text{ IS ON } L(p_j \theta_k)}\ dx\,dy$$

THE RADON TRANSFORM

$$g(p,\theta) = \int_y \int_x f(x,y)\, \delta(x\cos\theta + y\sin\theta - p)\, dx\, dy$$

JOHANN RADON, 1917

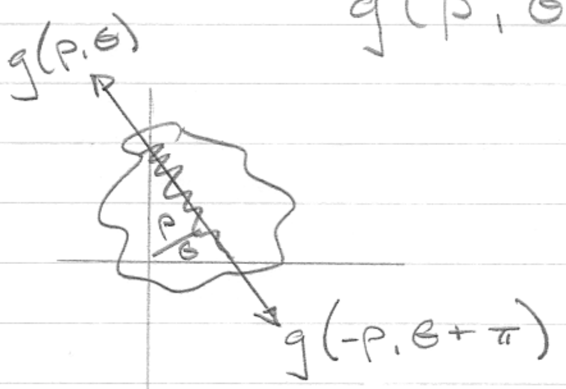
SINOGRAM

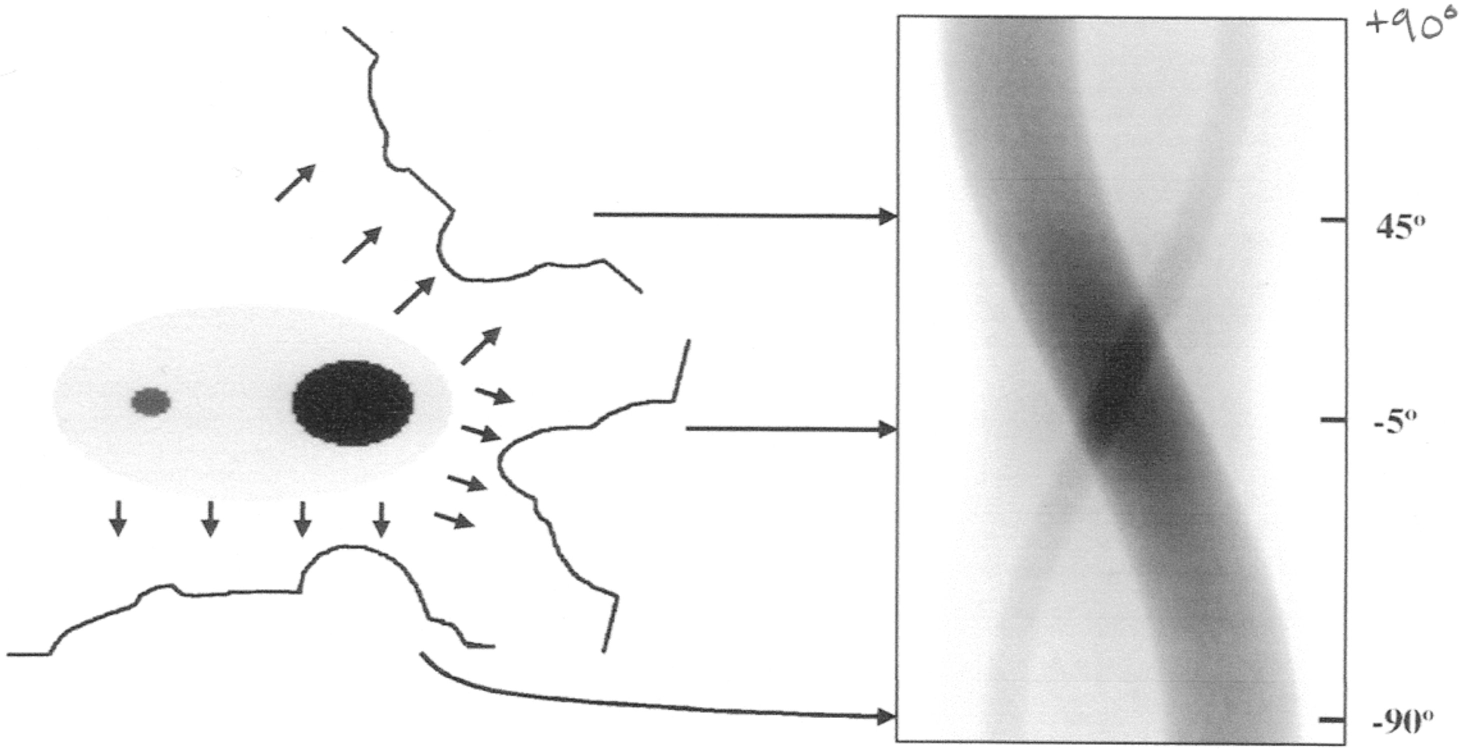
SHOW $g(p,\theta)$ FOR $p \in \{-R, +R\}$

$\theta \in \{0, \pi\}$

NOT 2π SINCE

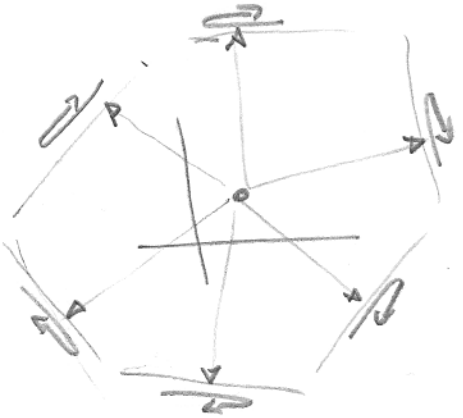
$$g(p,\theta) = g(-p, \theta+\pi)$$

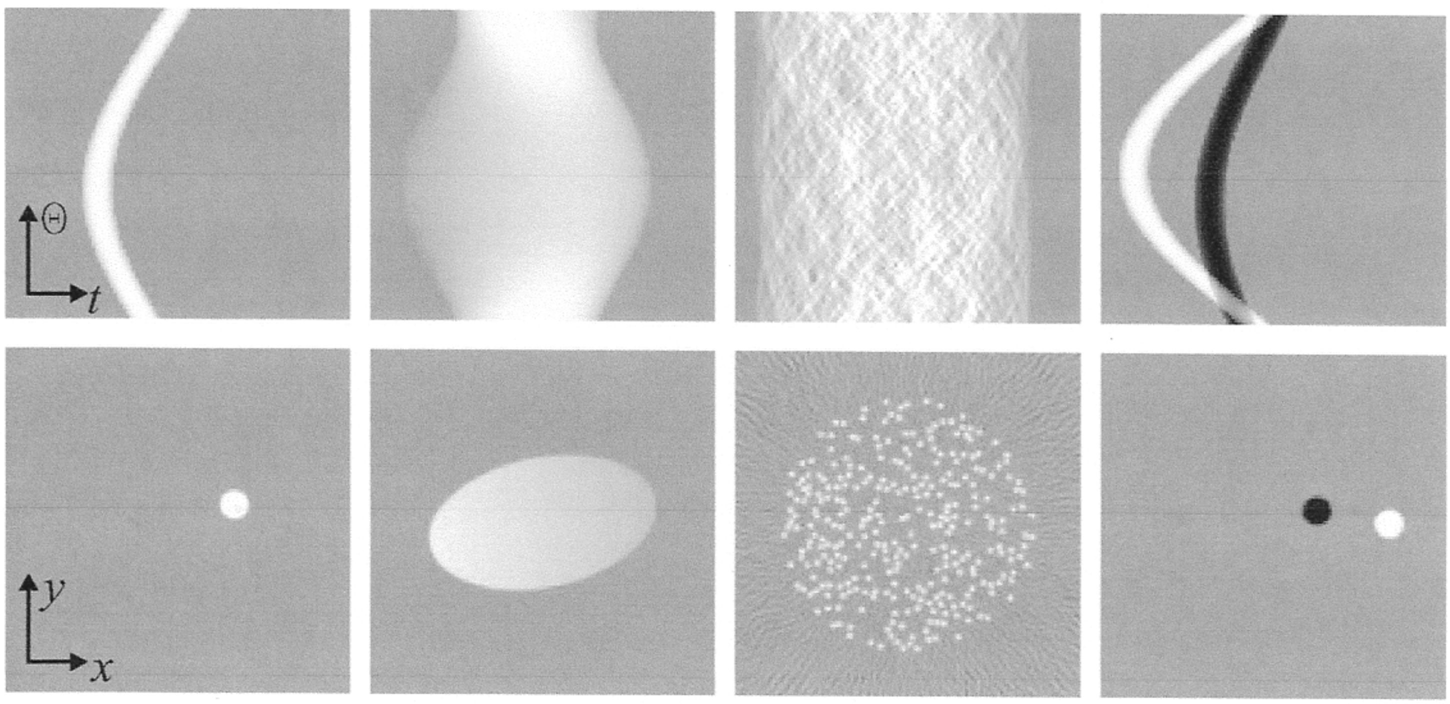




[Journal of Nuclear Medicine Technology]

SINOGRAM OF A POINT





[DONATH, BEERMANN, SCHREYER, J OPT SOC AMERICA,
23(5): 1048-1056, 2006]

* SO EACH POINT IN OBJECT CONTRIBUTES
A SINUSOID, WEIGHTED BY THE
ATTENUATION AT THE POINT

[WHAT IS SINOGRAM OF A LINE]